Sequences with identical autocorrelation spectra

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In this talk we explore sequences and their autocorrelation functions. Knowing the autocorrelation function of a sequence is equivalent to knowing the magnitude of its Fourier transform. Resolving the lack of phase information is called the phase problem. We say that two sequences are equicorrelational to mean that they have the same aperiodic autocorrelation function. In this paper, we investigate the necessary and sufficient conditions for two sequences to be equicorrelational, where we take into consideration the alphabet from which their terms are drawn. There are trivial forms of equicorrelationality arising from modifications that predictably preserve the autocorrelation, for example, negating sequences or writing the sequence in reverse order and then complex conjugating every term. By an exhaustive search of binary sequences up to length 43, we find that nontrivial equicorrelationality among binary sequences does occur, but is rare. We say that a positive integer n is *unequivocal* to mean that there is no pair of nontrivially equicorrelational binary sequences of length n; otherwise n is *equivocal*. For integers $n \leq 43$, we found that the unequivocal ones are 1–8, 10, 11, 13, 14, 19, 22, 23, 26, 29, 37, and 38. We prove that any multiple of a equivocal number is also equivocal, and pose open questions as to whether there are finitely or infinitely many unequivocal numbers and whether the probability of nontrivial equicorrelationality occurring tends to zero as the sequence length tends to infinity. (This is joint work with Adeebur Rahman and Michael J Ward.)