Secondary plateaued Boolean functions through addition of indicators

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Overview:

- Relevant definitions and notations
- Bent functions
- Plateaued functions
- Plateauedness of $f \oplus 1_R$ when f is plateaued
- Optimal plateaued functions in the $\mathcal{GMM}$ class

Relevant definitions and notations (I)

- $\mathbb{F}_2$ - the finite field with two elements, i.e. take $\{0,1\}$, add mod 2 and multiply as usual, example $1+1=0$, $1 \cdot 0 = 0, \ldots$
- $\mathbb{F}_2^n$ - n-dimensional vector space over $\mathbb{F}_2$.
  ex. $(1,0,1)+(1,0,0)=(0,0,1)$
- A Boolean function is any mapping from $\mathbb{F}_2^n \rightarrow \mathbb{F}_2$.
  (ex. $f(1,0,1)=0, f(1,0,0)=1, \ldots$)
- The set of all Boolean functions in n variables is denoted by $\mathcal{B}_n$.

Relevant definitions and notations (II)

- Walsh Hadamard transform:

$$W_f(u) = \sum_{x \in \mathbb{F}_2^n} (-1)^{f(x) \oplus u \cdot x}, \quad \text{for every } u \in \mathbb{F}_2^n$$

- Parseval's Relation: For every n-variable Boolean function f, we have

$$\sum_{v \in \mathbb{F}_2^n} W_f(v)^2 = 2^{2n}$$

- Walsh Support:

$$S_f = \{\omega \in \mathbb{F}_2^n : W_f(\omega) \neq 0\}$$

Bent functions

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- The $\mathcal{C}$ class of bent functions contains all the functions of the form

$$f(x, y) = x \cdot \pi(y) \oplus 1_{L^\perp}(x),$$

where $x, y \in \mathbb{F}_2^n$ and L is linear subspace of $\mathbb{F}_2^n$ and π is permutation on $\mathbb{F}_2^n$ such that $\phi(a + L)$ is a flat, for all $a \in \mathbb{F}_2^n$, where $\phi := \pi^{-1}$.

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- The class $\mathcal{D}$ of be bent functions is defined as

$$f(x, y) = x \cdot \pi(y) \oplus 1_{E_1}(x) 1_{E_2}(y),$$

where π is permutation on $\mathbb{F}_2^n$ and E_1, E_2 be two linear subspaces of $\mathbb{F}_2^n$ such that $\pi(E_2) = E_1^\perp$.

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$$W_f(u) \in \{0, \pm 2^{\frac{n+s}{2}}\}, \text{ for every } u \in \mathbb{F}_2^n,$$

where $s \geq 1$ if n is odd and $s \geq 2$ if n is even(s and n always have the same parity).

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- The $\#S_f$ of any s -plateaued function is 2^{n-s}.
- Semibent function: 1-plateaued or 2-plateaued function are semibent.

Addition of indicator to any f

The indicator of $R \subset \mathbb{F}_2^n$: $1_R(x) = 1$ **IFF** $x \in R$

Addition of indicator of R to $f : \mathbb{F}_2^n \to \mathbb{F}_2$, then WHT of $f \oplus 1_R$:

$$\begin{aligned} W_{f \oplus 1_R}(u) &= \sum_{x \in \mathbb{F}_2^n} (-1)^{f(x) \oplus 1_R(x) \oplus u \cdot x} \\ &= \sum_{x \in \mathbb{F}_2^n \setminus R} (-1)^{f(x) \oplus u \cdot x} + \sum_{x \in R} (-1)^{f(x) \oplus 1_R(x) \oplus u \cdot x} \\ &= \sum_{x \in \mathbb{F}_2^n} (-1)^{f(x) \oplus u \cdot x} - 2 \sum_{x \in R} (-1)^{f(x) \oplus u \cdot x} \\ &= W_f(u) - 2 \sum_{x \in R} (-1)^{f(x) \oplus u \cdot x} = W_f(u) - 2U(u). \end{aligned} \quad (1)$$

Plateauedness of $f \oplus 1_R$

Lemma (E. Pasalic, S.Hodžic, S. Kudin, D.A.Khan; BFA 2024)

Let $f : \mathbb{F}_2^n \rightarrow \mathbb{F}_2$. Then f is s-plateaued $(1 \leq s \leq n)$ if and only if it holds that $\#S_f = 2^{n-s}$ and

$$\begin{cases} W_f(u) = 0, & u \notin S_f, \\ W_f(u) \equiv 2^{\frac{n+s}{2}} \ (mod\ 2^{\frac{n+s}{2}+1}), & u \in S_f. \end{cases} \quad (2)$$

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$$2^{2n} = \sum_{u \in \mathbb{F}_2^n} W_f^2(u) \geq \#S_f \cdot 2^{n+s} = 2^{n-s} \cdot 2^{n+s} = 2^{2n},$$

i.e. $W_f^2(u) = 2^{n+s}$, or $W_f(u) = \pm 2^{\frac{n+s}{2}}, \forall u \in S_f$.

Hence, $f \oplus 1_R$ is s- plateaued function.

$\mathcal{GMM}_{\frac{n}{2}+k}$ Class

The Maiorana-McFarland class $\mathcal{M}$ is the set of m-variable ($m = 2n$) Boolean functions of the form

$$f(x, y) = x \cdot \pi(y) + g(y), \quad \forall x, y \in \mathbb{F}_2^n,$$

where π is a permutation on $\mathbb{F}_2^n$ and $g \in \mathcal{B}_n$.

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Definition

The set of all Boolean functions $f_{\frac{n+k}{2}} : \mathbb{F}_2^{\frac{n+k}{2}} \times \mathbb{F}_2^{\frac{n-k}{2}} \to \mathbb{F}_2$, of the form

$$f_{\frac{n+k}{2}}(x, y) = x \cdot \phi^{(k)}(y) \oplus g_k(y), \;\; x \in \mathbb{F}_2^{\frac{n\pm k}{2}}, y \in \mathbb{F}_2^{\frac{n\mp k}{2}},$$

is called $\mathcal{GMM}_{\frac{n+k}{2}}$ class, where $\phi^{(k)} : \mathbb{F}_2^{\frac{n\mp k}{2}} \to \mathbb{F}_2^{\frac{n\pm k}{2}}$ and $g_k \in \mathcal{B}_{\frac{n\mp k}{2}}$, for $0 \leq k < n$. For $k = 0$ this class corresponds to the $\mathcal{MM}$ class of bent functions when $\phi^{(0)}$ is a permutation on $\mathbb{F}_2^{\frac{n}{2}}$.

Towards optimal plateaued functions

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Lemma (E. Pasalic, S.Hodžic, S. Kudin, D.A.Khan; BFA 2024)

Let $\phi : \mathbb{F}_2^t \to \mathbb{F}_2^{t+j}$ be defined as $\phi = (\pi(y), g_1(y), \ldots, g_j(y))$ so that at least one of g_j has degree t and π is a permutation on $\mathbb{F}_2^t$. Then, ϕ is injective and of maximum degree t.

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Sketch of proof:

- If for some $y \neq y' \in \mathbb{F}_2^t$, we have $\phi(y) = \phi(y')$, $\implies \pi(y) = \pi(y')$. A contradiction as π is a permutation, Hence, ϕ is injective.
- At least one of g_j has maximum algebraic degree t, so does ϕ.

Optimal plateaued functions in $\mathcal{GMM}_{\frac{n}{2}+k}$ class

Theorem 1 (E. Pasalic, S.Hodžic, S. Kudin, D.A.Khan; BFA 2024)

Let $f(x,y) = x \cdot \phi(y) + h(y)$, *where* $x \in \mathbb{F}_2^{\frac{n+k}{2}}, y \in \mathbb{F}_2^{\frac{n-k}{2}}$, *for* $0 < k < n$.
Let $\phi(y) = (\pi(y), g_1(y), \cdots, g_k(y))$, *where*

- π *is permutation on* $\mathbb{F}_2^{\frac{n-k}{2}}$,
- $g_1, \ldots, g_k \in \mathcal{B}_{\frac{n-k}{2}}$ *be such that* $\max_i \deg(g_i) = \frac{n-k}{2}$,
- $h \in \mathcal{B}_{\frac{n-k}{2}}$ *is arbitrary.*

Then, $f(x,y) = x \cdot \phi(y) + h(y)$ *is an optimal k-plateaued function.*

Linear structures

An element $a \in F_2^n$ is called a linear structure of $f \in \mathcal{B}_n$, if

$$D_a f = f(x+a) + f(x) = constant \quad \forall x \in \mathbb{F}_2^n.$$

$f \in \mathcal{B}_n$ has no linear structures, if 0_n is the only linear structure of f.

Theorem 2 (E. Pasalic, S.Hodžic, S. Kudin, D.A.Khan; BFA 2024)

Let f be defined as in Theorem 1 and assume that $D_b\phi(y) \neq 0_{n/2+k}$ and $a \cdot \phi(y) \neq 0$. Then, f has no linear structures.

Sketch of proof

- The function f has no linear structures if

$$D_{a,b}f(x,y) \neq \textit{constant}, \quad \textit{where} \quad (a,b) \in \mathbb{F}_2^{\frac{n+k}{2}} \times \mathbb{F}_2^{\frac{n-k}{2}}.$$

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- The derivative of $f(x,y)$ is given as:

$$D_{(a,b)}f(x,y) = x \cdot D_b\phi(y) + a \cdot \phi(y+b) + D_b h(y)$$

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- **If** $b = 0$ then, $D_{(a,b)}f(x,y) \neq 0 \iff a \cdot \phi(y) \neq 0$
- **If** $b \neq 0$ then, sufficient condition for $D_{(a,b)}f(x,y) \neq 0$ is $D_b\phi(y) \neq 0$.

Addition of an indicator depending on both x and y

Theorem (E. Pasalic, S.Hodžic, S. Kudin, D.A.Khan; BFA 2024)

Let $\pi : \mathbb{F}_2^{n/2} \to \mathbb{F}_2^{n/2}$ be a permutation, with n even. Suppose that

$A = a + E$ be an affine subspace of $\mathbb{F}_2^{n/2}$, $dim(A) = n/2 - 1$, and $B \subset \mathbb{F}_2^{n/2}$ with $\#B = 2$. For $g(x, y) = x \cdot \pi(y) \oplus 1_{A \times B}(x, y)$ it holds that:

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i) If $\#(B \cap \pi^{-1}(u \oplus E^{\perp})) \in \{0, 2\}$ for all $u \in \mathbb{F}_2^{n/2}$, then g is bent.

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i) If $\#(B \cap \pi^{-1}(u \oplus E^{\perp})) \in \{0, 2\}$ for all $u \in \mathbb{F}_2^{n/2}$, then g is bent.

ii) If $\#(B \cap \pi^{-1}(u \oplus E^{\perp})) \in \{0, 1\}$ for all $u \in \mathbb{F}_2^{n/2}$, then g is semi-bent.

Addition of an indicator depending on both x and y

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2. If $\#(B \cap \pi^{-1}(u \oplus E^{\perp})) \in \{0,1\}$ for all $u \in \mathbb{F}_2^{n/2}$, then g is semi-bent.
3. If $\#(B \cap \pi^{-1}(u \oplus E^{\perp})) \in \{0,1,2\}$ for all $u \in \mathbb{F}_2^{n/2}$, then g is 5-valued spectra function.

Lemma 3 (E. Pasalic, S.Hodžic, S. Kudin, D.A.Khan; BFA 2024)

Let $f : \mathbb{F}_2^n \to \mathbb{F}_2$ *be a k-plateaued function,* $0 \leq k \leq n$, $n \equiv k \pmod 2$,

and let V *be a subspace of* $\mathbb{F}_2^n$ *with* $\dim(V) = \frac{n+k}{2}$.

- *If* $f(v) = 0$, *for all* $v \in V$, *then* $W_f(w) = 2^{\frac{n+k}{2}}$, *for all* $w \in V^{\perp}$.
- *If* $f(v) = 1$, *for all* $v \in V$, *then* $W_f(w) = -2^{\frac{n+k}{2}}$, *for all* $w \in V^{\perp}$.

Theorem 4 (E. Pasalic, S.Hodžic, S. Kudin, D.A.Khan; BFA 2024)

Let $g : \mathbb{F}_2^n \to \mathbb{F}_2$ *be a* k*-plateaued function,* $0 \le k \le n$*,* $n \equiv k \pmod 2$*, and let* V *be a subspace of* $\mathbb{F}_2^n$*,* $\dim(V) = \frac{n+k}{2}$*, such that* g *is constant on* V*. Then, the function* $f = g + 1_V$ *is also a* k*-plateaued function.*

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Sketch of proof:

- $g(v) = 0$, for all $v \in V$,

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Sketch of proof:

- $g(v) = 0$, for all $v \in V$,(Proof is analogous: $g(v) = 1$).

$$W_f(a) = \sum_{x \in \mathbb{F}_2^n} (-1)^{f(x)+x\cdot a} = \sum_{x \in \mathbb{F}_2^n} (-1)^{g(x)+x\cdot a} - 2\sum_{v \in V} (-1)^{g(v)+v\cdot a}$$
$$= W_g(a) - 2\sum_{v \in V} (-1)^{v\cdot a} = W_g(a) - 2(2^{\frac{n+k}{2}})1_{V^\perp}(a). \tag{3}$$

- For $a \in \mathbb{F}_2^n \setminus V^\perp$, we have $1_{V^\perp}(a) = 0 \implies W_f(a) \in \left\{0, \pm 2^{\frac{n+k}{2}}\right\}$

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- For $a \in V^\perp$, from Lemma 3, we have $W_g(a) = 2^{\frac{n+k}{2}}$, and from Equation (3) we get

$$W_f(a) = 2^{\frac{n+k}{2}} - 2^{\frac{n+k}{2}+1} = -2^{\frac{n+k}{2}}.$$

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- We conclude that $W_f(a) \in \left\{0, \pm 2^{\frac{n+k}{2}}\right\}$, for all $a \in \mathbb{F}_2^n$, hence f is a k-plateaued function.

Class $\mathcal{D}$ of plateaued functions

Corollary 1 (E. Pasalic, S.Hodžic, S. Kudin, D.A.Khan; BFA 2024)

Let $g(x, y) = x \cdot \phi(y)$ be any k-plateaued function in $\mathcal{GMM}_{\frac{n+k}{2}}$ class, where $x \in \mathbb{F}_2^{\frac{n+k}{2}}$, $y \in \mathbb{F}_2^{\frac{n-k}{2}}$ and the mapping $\phi : \mathbb{F}_2^{\frac{n-k}{2}} \to \mathbb{F}_2^{\frac{n+k}{2}}$ for $0 < k < n$. Let $E = E_1 \times E_2$ be a linear subspace of $\mathbb{F}_2^{\frac{n+k}{2}} \times \mathbb{F}_2^{\frac{n-k}{2}}$, where E_1 and E_2 are subspaces of $\mathbb{F}_2^{\frac{n+k}{2}}$ and $\mathbb{F}_2^{\frac{n-k}{2}}$ respectively, such that $\phi(E_2) = E_1^{\perp}$ and $\dim(E) = \frac{n+k}{2}$. Then, $f(x, y) = x \cdot \phi(y) \oplus 1_{E_1}(x)1_{E_2}(y)$ is a k-plateaued.

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Remark:

- Very similar conditions as for Carlet's class $\mathcal{D}$ of bent functions.
- Research task is obvious going outside $\mathcal{GMM}_{(n+k)/2}$.

Thank you