Constructing designs using functions

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Abstract

Both planar functions and APN functions have been used to construct designs; projective planes in the former case, and semibiplanes in the latter. While both are often defined in terms of an optimal (read minimal) differential uniformity, there is another way to make them part of a uniform definition which generalizes the previous design constructions as well.

Let $\mathcal{G}, \mathcal{H}$ be finite abelian groups, written additively, and let λ be a proper positive divisor of the order of $\mathcal{G}$. A function $f : \mathcal{G} \to \mathcal{H}$ is a *semiplanar function of index* λ if for every nonidentity $a \in \mathcal{G}$ and any $b \in \mathcal{H}$, the equation

$$f(x + a) - f(x) = b$$

has either 0 or λ solutions in x. In the case where the orders of $\mathcal{G}$ and $\mathcal{H}$ are equal, $\lambda = 1$ corresponds to the planar function case, and $\lambda = 2$ corresponds to the APN function case. The same incidence structure used previously to construct projective planes from planar functions or semibiplanes from APN functions can be used to construct semisymmetric designs from semiplanar functions.

Of course, nothing is free – plenty of problems arise. We will address what are probably the main two: whether semiplanar functions exist for $\lambda > 2$, and the connectivity of the design. There are a couple of surprises...