Radboud University

On round functions of permutations

Joan Daemen, Radboud University NL

BFA, September 7, 2023

Voss, Norway

ESCADA

Permutation-based cryptography

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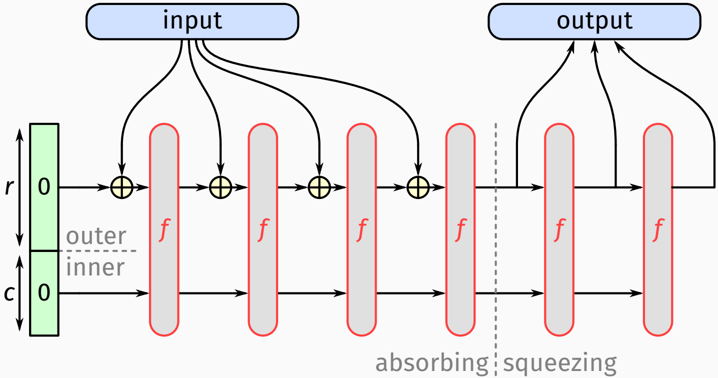

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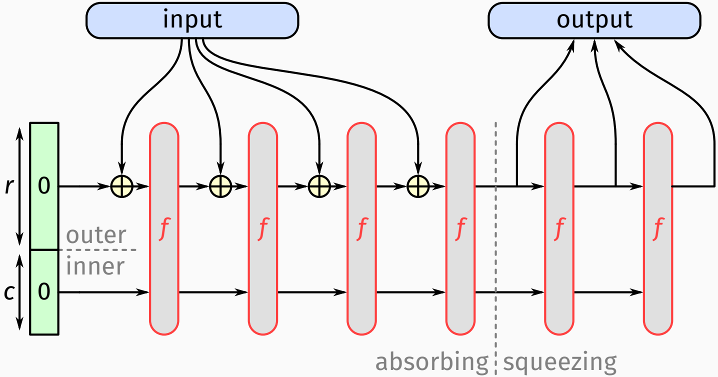
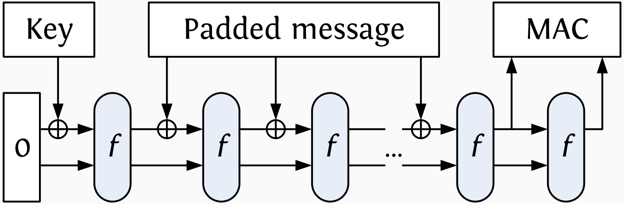
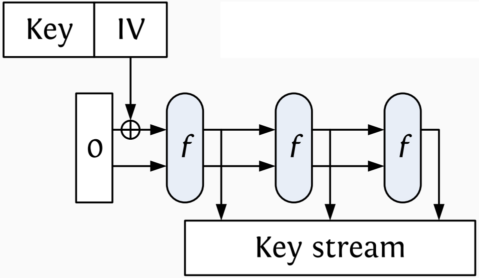


A priori for unkeyed hashing

Mac computation with sponge



Stream encryption with sponge



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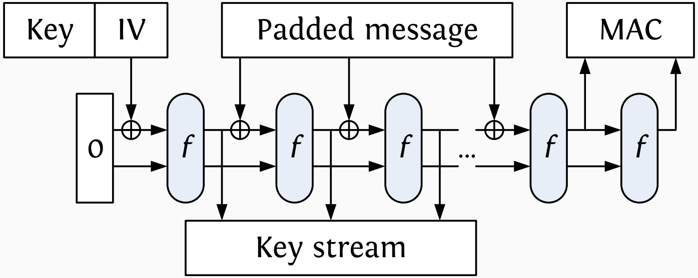

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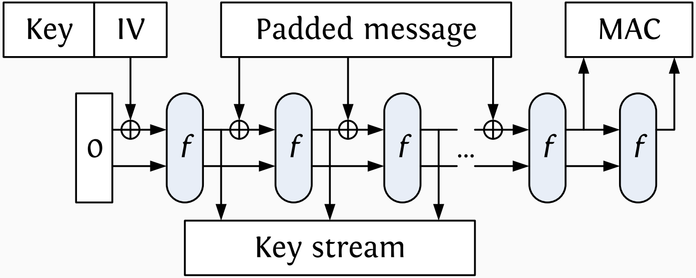

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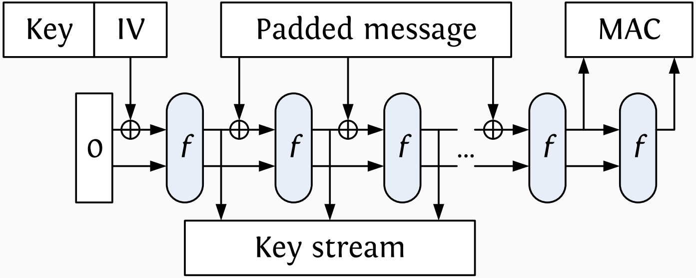

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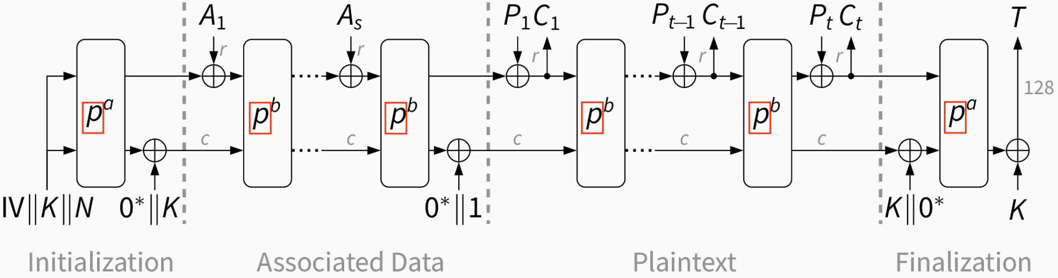
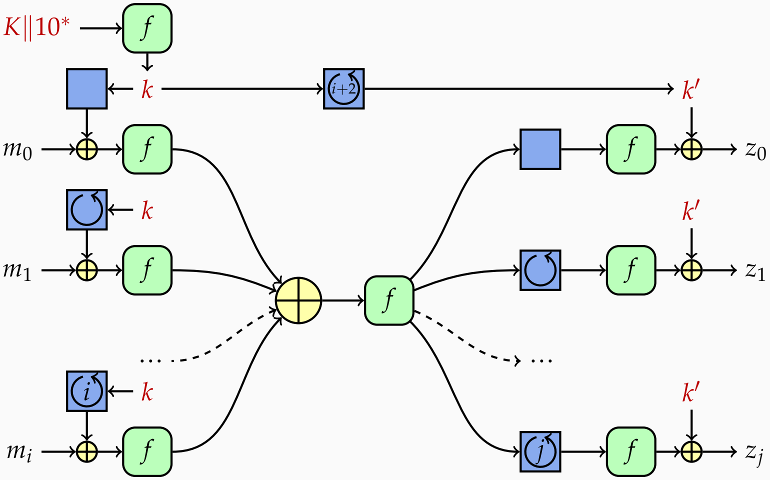
Especially the variant MonkeyDuplex that we proposed in [Bertoni et al., DIAC 2012]

Ascon [Dobraunig et al. 2016]



NIST's new standard for lightweight authenticated encryption!



$K\|10^*$
f
k
$i+2$
k'
m_0
f
f
z_0
k
k'
m_1
f
f
f
z_1
$\cdots$
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i
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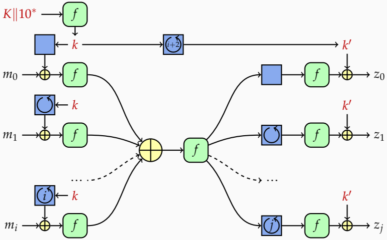
Farfalle construction [Bertoni et al., 2017]



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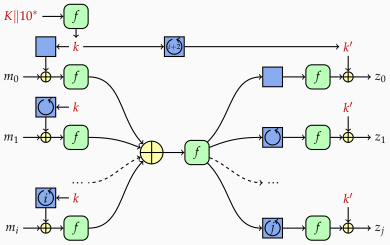

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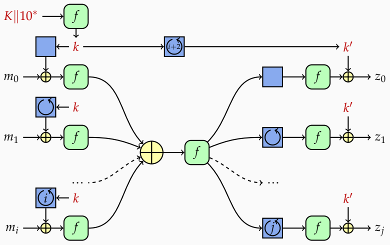

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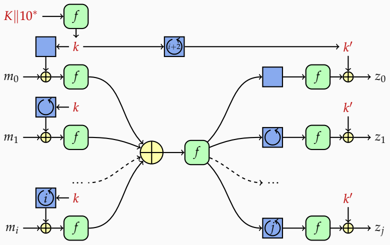

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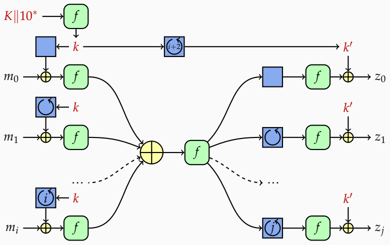

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  - Competitive with AES even on CPUs with AES-NI instruction

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- Interesting hardware benchmarks related to lightweight:
  - https://eprint.iacr.org/2020/1207
  - https://eprint.iacr.org/2020/1459
  - https://eprint.iacr.org/2021/049

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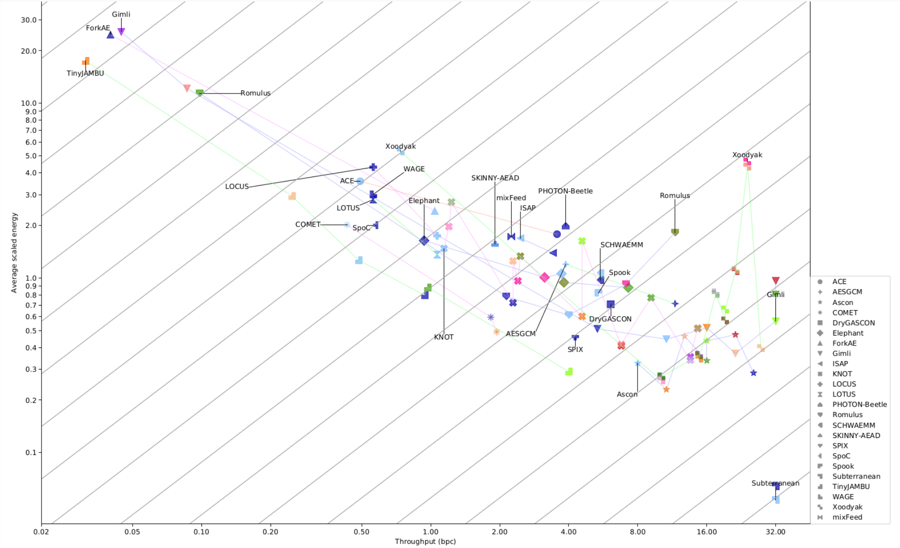

Focus on the permutation

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  - suitability for masking: low algebraic degree building blocks
- As opposed to block ciphers: no need for efficient inverse

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LC DC requirements are of the following type:

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  - or to harvest (linear) equations in unknown state bits

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We can further split $\mathsf{DP}_{\gamma}(b^0, a^1)$ with $b^0 = (b_0, b_1, \ldots b_{m-1}), a^1 = (a_0, a_1, \ldots a_{m-1})$

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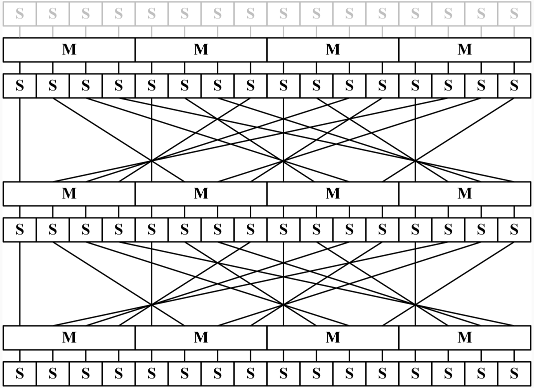
  - less *corners where weaknesses can hide*

Example of cell-oriented wide trail design: Rijndael-128 [Daemen & Rijmen, 1998]



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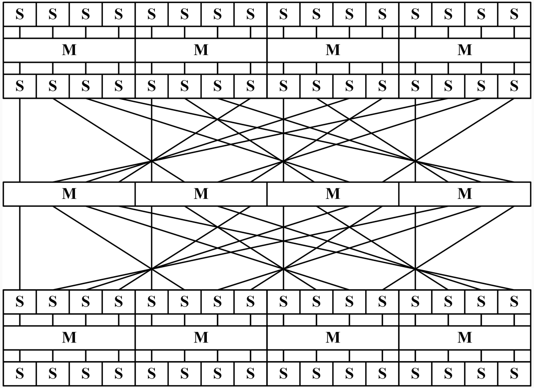

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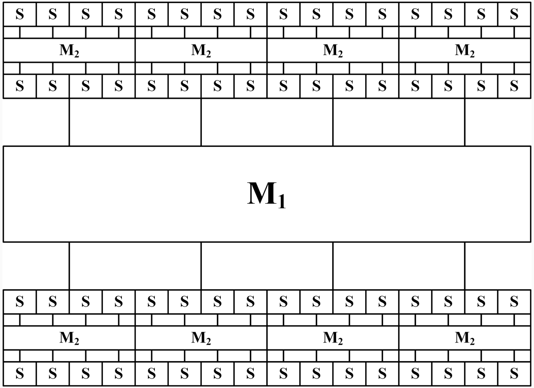

Thanks to *superboxes* proving any 4-round trail has at least 25 active S-boxes is easy!

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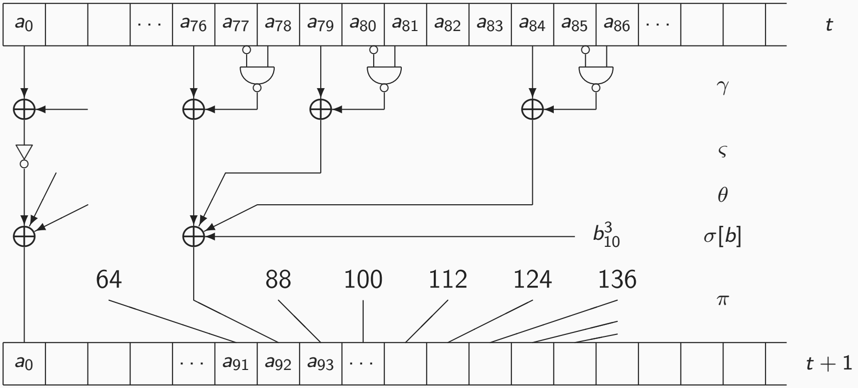

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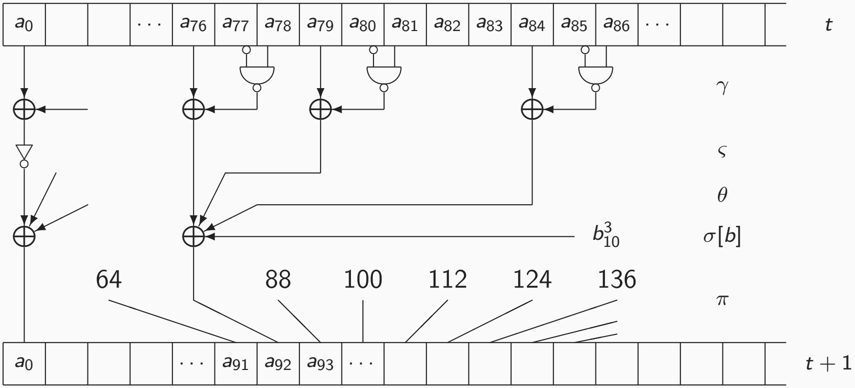

Proving trail bounds requires computer-assisted search

Choice of the S-box

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Correlation and LP of linear approximations (a, b) of a transformation of $\mathbb{F}_{p^n}$

$$\mathrm{C}(a, b) = \frac{\sum_x \omega^{\mathrm{Tr}(ax - bf(x))}}{p^n} \text{ with } \omega = e^{\frac{2\pi i}{p}}$$

$$\mathrm{LP}(a, b) = \mathrm{C}(a, b)\overline{\mathrm{C}}(a, b)$$

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Transformations in $\mathbb{F}_{p^n}$ with much symmetry: power functions

- Functions of the form $y \leftarrow x^e$
- Invertible if e is coprime to $p^n - 1$
- Invertible power functions form a group
  - isomorphic to $(\mathbb{Z}/(p^n - 1)\mathbb{Z})^*$
  - order is $\varphi(p^n - 1)$
- Inverse of $y \leftarrow x^e$ is $y \leftarrow x^d$ with $d = e^{-1} \bmod (p^n - 1)$

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- Multiplicative inverse mapping is often called the Kaisa S-box [Nyberg, EC '93]

DP-table (aka scaled DDT) of $y \leftarrow x^{-1}$ in $\mathbb{F}_2^4$

1/8	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
1	2					1	1		1		1		1	1	
2			1				1		2	1		1	1		1
3		1					1	1		1	1	1		2	
4					1	1		1		1			2	1	1
5				1		1		1	1		2	1			1
6	1			1	1	1	2			1		1			
7	1	1	1			2	1	1							1
8			1	1	1		1				1			1	2
9	1	2			1				1			1		1	1
a		1	1	1		1						2	1	1	
b	1		1		2			1			1	1	1		
c		1	1		1	1			1	2	1				
d	1	1		2						1	1		1		1
e	1		2	1				1	1	1				1	
f		1		1	1		1	2	1				1		

DP-table of $x \leftarrow x^{-1}$ in $\mathbb{F}_2^4$, reordered

1/8	1	α	α^2	α^3	α^4	α^5	α^6	α^7	α^8	α^9	α^{10}	α^{11}	α^{12}	α^{13}	α^{14}
1	2					1		1			1	1		1	1
α					1		1			1	1		1	1	2
α^2				1		1			1	1		1	1	2	
α^3			1		1			1	1		1	1	2		
α^4		1		1			1	1		1	1	2			
α^5	1		1			1	1		1	1	2				
α^6		1			1	1		1	1	2					1
α^7	1			1	1		1	1	2					1	
α^8			1	1		1	1	2					1		1
α^9		1	1		1	1	2					1		1	
α^{10}	1	1		1	1	2					1		1		
α^{11}	1		1	1	2					1		1			1
α^{12}		1	1	2					1		1			1	1
α^{13}	1	1	2					1		1			1	1	
α^{14}	1	2					1		1			1	1		1

Correlation matrix (aka scaled LAT) of $x \leftarrow x^{-1}$ in $\mathbb{F}_2^4$

1/4	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
1	-1		1		1		-1	-1		-1	2	1		1	2
2				1	1	-1	-1		2	2		1	-1	1	-1
3	1		-1	-1	2	1	2	-1		1			-1		1
4		1	-1			1	-1			1	-1	2	2	-1	1
5	1	1	2		-1	1		-1	2		-1	-1			1
6		-1	1	1	1	2		2		-1	-1	1	-1		
7	-1	-1	2	-1			1	1		2	1		1	-1	
8	-1		-1		-1	2	1		1		1		1	2	-1
9		2			2			1	1	-1	1	-1	1	-1	-1
a	-1	2	1	1		-1	2		-1		-1	1		1	
b	2			-1	-1	-1	1	1	1	-1	1	2			
c	1	1		2	-1	1			-1	1	2		-1	-1	
d		-1	-1	2		-1	1	1	1			-1	1		2
e	1	1		-1			-1	2	-1	1		-1		2	1
f	2	-1	1	1	1			-1	-1				2	1	-1

Correlation matrix of $x \leftarrow x^{-1}$ in $\mathbb{F}_2^4$, reordered

1/4	1	β	β^2	β^3	β^4	β^5	β^6	β^7	β^8	β^9	β^{10}	β^{11}	β^{12}	β^{13}	β^{14}
1		-1	-1	1	-1	2	1		-1	1	2		1		
β	-1	-1	1	-1	2	1		-1	1	2		1			
β^2	-1	1	-1	2	1		-1	1	2		1				-1
β^3	1	-1	2	1		-1	1	2		1				-1	-1
β^4	-1	2	1		-1	1	2		1				-1	-1	1
β^5	2	1		-1	1	2		1				-1	-1	1	-1
β^6	1		-1	1	2		1				-1	-1	1	-1	2
β^7		-1	1	2		1				-1	-1	1	-1	2	1
β^8	-1	1	2		1				-1	-1	1	-1	2	1	
β^9	1	2		1				-1	-1	1	-1	2	1		-1
β^{10}	2		1				-1	-1	1	-1	2	1		-1	1
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    - DP table has 16 non-zero entries per row, correlation matrix 26

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- When choosing the normal basis, $\mathrm{Tr}(ab) = 1$ translates to $a_0 b_0 + a_1 b_1 + a_2 b_2 = 1$

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  - $y_i \leftarrow x_i + (x_{i+1 \bmod 3} + 1)x_{i+2 \bmod 3}$: costs 1 xor and 1 and per bit

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- Excellent trade-off between implementation cost and non-linearity

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- Shuffle moving bytes of a column to different columns: here transposing the array
- Easy to prove that any 4-round trail has at least 25 active S-boxes

Generalizations of and advances in the square approach

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Function	shape	cells	width	type
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ECHO [Benadjila et al., 2008]	4^4	bytes	2048	perm
JH [Wu, 2008]	2^8	nibbles	1024	perm
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- insight: cost increases sharply with MDS matrix dimension

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  - Two different shuffles in rounds with index $1 \bmod 4$ and $3 \bmod 4$
  - Any 8-round trail has at least $5^3 = 125$ active S-boxes

Showdown Saturnin vs AES (not counting round key addition)

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AES

# rounds	cost xor	cost and/or	min. trail weight
1	14	4	6
2	28	8	30
3	42	12	56
4	56	16	150

Saturnin

# rounds	cost xor	cost and/or	min. trail weight
1	3,75	1,5	2
2	7,5	3	10
3	11,25	4,5	18
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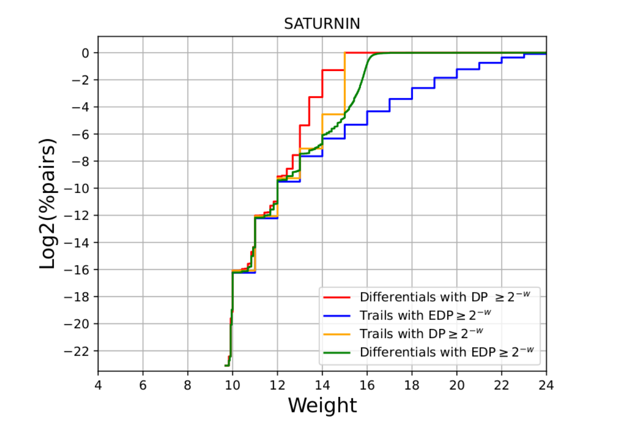
Clustering and *clipping* in the AES superbox:

- massive clustering of trails in differentials [Daemen & Rijmen, SCN 2006]
- clipping: DP(Q) strongly deviates from EDP(Q) for most trails [Daemen & Rijmen, IET 2007]

Clustering and clipping in the Saturnin superbox, illustrated

Much less clustering and clipping than in AES thanks to smaller S-box, still significant



graph courtesy of Giovanni Uchua de Assis

Bit-oriented mixing

- In general it is just a binary matrix $\mathbf{M}$
  - operating on the full state, or
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- Invertible if $\theta(X)$ is coprime to $1 + X^m$
- Often one takes a multiplication polynomial that is a trinomial
- Unless carefully chosen, inverse of $\theta(X)$ is dense
  - no problem if the inverse of the permutation is not needed
  - has an advantage for trail bounds

Some primitives using bit-oriented circulant mixing

Function	length	# t	non-lin.	b	shuffle
Cellhash [Daemen, AC 1991]	257	3	χ_{257}	257	multiplicative
3Way [Daemen, 1993]	12	7	χ_3	96	2 row shift steps
BaseKing [Daemen, 1994]	12	7	χ_3	192	2 row shift steps
Panama [Daemen & Clapp, 1997]	17	3	χ_{17}	544	1 row shift step
SHA-256 [NIST, 2001]	32	3	ARX	256	-
SHA-512 [NIST. 2001]	64	3	ARX	512	-
RadioGatun [Bertoni et al., 2006]	19	3	χ_{19}	608	1 row shift step
Ascon [Dobraunig et al., 2019]	64	3	χ_5+	320	different $m(x)$

Ascon-p Round function

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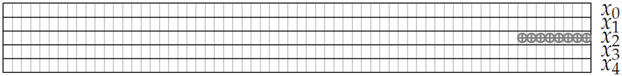
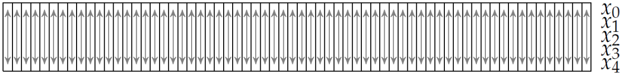
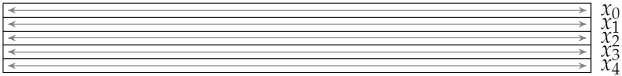
- 320-bit state: 5 rows $x_0, \ldots, x_4$ and 64 columns
- Round function $\mathrm{R} = p_L \circ p_S \circ p_C$



(a) Round constant addition p_C



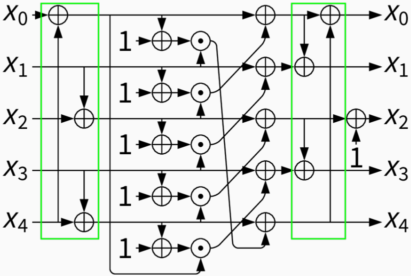
(b) Substitution layer p_S with 5-bit S-box $\mathcal{S}(x)$



(c) Linear layer with 64-bit diffusion functions $\Sigma_i(x_i)$

figure by Ascon team

Operations dedicated to mixing in Ascon-p



6 bitwise XOR

$$
\begin{aligned}
x_0 &\leftarrow x_0 \oplus (x_0 \ggg 19) \oplus (x_0 \ggg 28) \\
x_1 &\leftarrow x_1 \oplus (x_1 \ggg 61) \oplus (x_1 \ggg 39) \\
x_2 &\leftarrow x_2 \oplus (x_2 \ggg 1) \oplus (x_2 \ggg 6) \\
x_3 &\leftarrow x_3 \oplus (x_3 \ggg 10) \oplus (x_3 \ggg 17) \\
x_4 &\leftarrow x_4 \oplus (x_4 \ggg 7) \oplus (x_4 \ggg 41)
\end{aligned}
$$

10 bitwise XOR + 10 cyclic shifts

Showdown Ascon-p vs Saturnin (not counting round key/constant addition)

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Saturnin

# rounds	cost xor	cost and/or	min. trail weight
1	3,75	1,5	2
2	7,5	3	10
3	11,25	4,5	18
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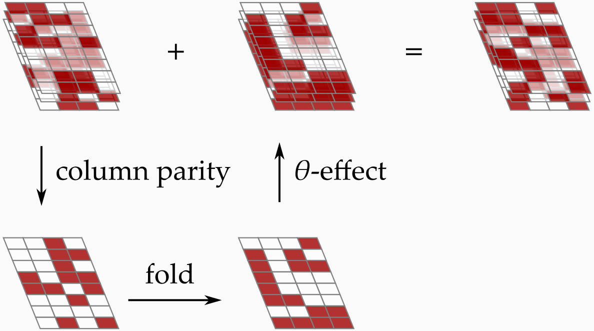
6	22,5	9	90
7	26,25	10,5	122
8	30	12	250

Ascon

# rounds	cost xor	cost and/or	min. trail weight diff	min. trail weight lin
1	4,2	1	2	2
2	8,4	2	8	8
3	12,6	3	40	28
4	16,8	4	≥ 86	≥ 88
5	21	5	≥ 100	≥ 96
6	25,2	6	≥ 129	≥ 132

Another type of mixing layer: column parity mixers (Keccak-f and Xoodoo)



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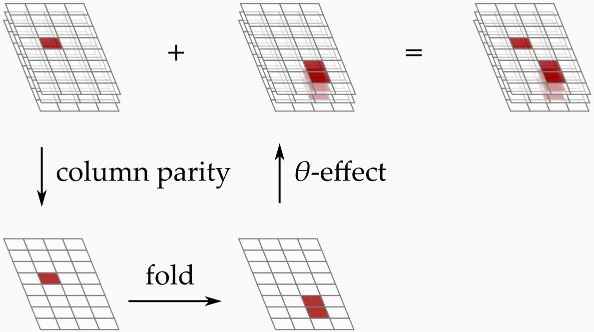

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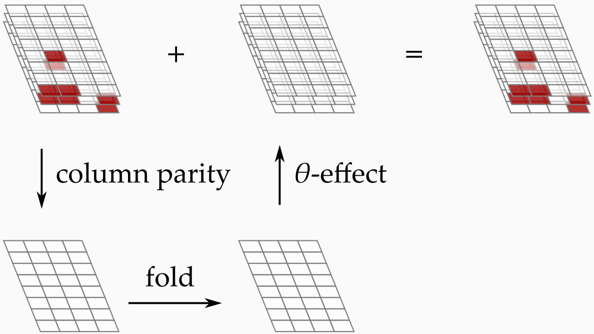

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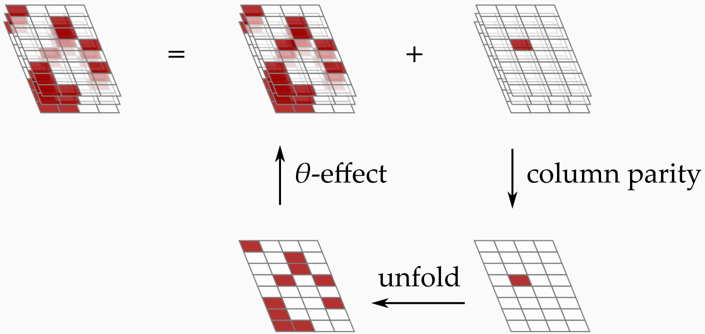

- Good average diffusion, identity for states in *kernel*
- Cost: 2 xor per bit

Showdown Xoodoo vs Ascon-p

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Xoodoo

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2	6	2	8
3	9	3	36
4	12	4	80
5	15	5	≥ 98
6	18	6	≥ 132

Trail clustering and independence of round differentials in Xoodoo

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  - for each differential trails we have: $\mathrm{DP}(Q) = \mathrm{EDP}(Q)$
- 4-round trails: work in progress
  - 4 trails of weight 80
  - 2 of these cluster into differential with $\mathrm{EDP}(a, b) = 2^{-79}$
  - dependence of round differentials: we're starting

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  - no noticeable dependencies between round differentials
  - attention for resistance against summation attacks

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  - shuffle with *as few shifts as we can afford*

Thanks for your attention!