

# Vectorial bent functions in odd characteristic and their components

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Let  $p$  be an odd prime and  $\mathbb{V}_n$  an  $n$ -dimensional vector space over  $\mathbb{F}_p$ . By a result of Kumar, Scholtz and Welsh 1986, for a bent function  $f : \mathbb{V}_n \rightarrow \mathbb{F}_p$  the *Walsh coefficient*  $\widehat{f}(b)$  at  $b \in \mathbb{V}_n$  of  $f$  always satisfies

$$\widehat{f}(b) = \begin{cases} \pm \epsilon_p^{f^*(b)} p^{n/2} & : p^n \equiv 1 \pmod{4}; \\ \pm i \epsilon_p^{f^*(b)} p^{n/2} & : p^n \equiv 3 \pmod{4}, \end{cases} \quad (1)$$

where  $f^*$  is a function from  $\mathbb{V}_n$  to  $\mathbb{F}_p$ , called the dual of  $f$ . A bent function  $f : \mathbb{V}_n \rightarrow \mathbb{F}_p$  is called *weakly regular* if, for all  $b \in \mathbb{V}_n$ , we have  $\widehat{f}(b) = \zeta \epsilon_p^{f^*(b)} p^{n/2}$  for some  $\zeta \in \{\pm 1, \pm i\}$ , cf. Equation (1). If  $\zeta = 1$  we call  $f$  *regular*. If (the sign of)  $\zeta$  changes with  $b \in \mathbb{V}_n$ , then  $f$  is called *non-weakly regular bent*. Weakly regular bent functions  $f$  belong to the class of *dual-bent functions*, for which the dual  $f^*$  is bent as well. A non-weakly regular bent function can be either dual-bent or *non-dual-bent*.

The classical examples of bent functions are all weakly regular. In particular all known infinite classes of bent monomials and binomials are weakly regular, the components of the vectorial Maiorana-McFarland and the vectorial spread bent functions are regular, and the components of all known planar functions are weakly regular (all of which besides from the Coulter-Matthews function are quadratic hence necessarily weakly regular). The first construction of non-weakly regular though dual bent functions is given in [1], recently the first construction of non-dual bent functions was presented in [2].

We first show that many infinite classes of  $p$ -ary bent monomials and binomials in the literature are actually vectorial constructions. In the second part we give a positive answer to the question if non-weakly regular bent functions can be components of a vectorial bent function. We present the first construction of vectorial bent functions of which the components are non-weakly regular but dual-bent, and the first construction of vectorial bent functions with non-dual bent components.

## References

- [1] A. Çeşmelioglu, G. McGuire, W. Meidl, A construction of weakly and non-weakly regular bent functions, *J. Comb. Theory, Series A* 119 (2012), 420–429.
- [2] A. Çeşmelioglu, W. Meidl, A. Pott, There are infinitely many bent functions for which the dual is not bent. *IEEE Trans. Inform. Theory* 62 (2016), 5204–5208.