Table 1: CCZ-equivalences of Families of APN Polynomials over $\mathbb{F}_{2^n}$ from table 2 ($6 \le n \le 11$, u is primitive in $\mathbb{F}_{2^n}^*$)

N°	Functions*	$n=6$	$n=7$	$n=8$	$n=9$	$n=10$	$n=11$
1	$x^{2^s+1} + u^{2^k-1} x^{2^{ik}+2^{mk+s}}$, $p=3$	Gold	–	–	–	–	–
2	$x^{2^s+1} + u^{2^k-1} x^{2^{ik}+2^{mk+s}}$, $p=4$	–	–	–	–	–	–
3	$x^{2^{2i}+2^i} + bx^{q+1} + cx^{q(2^{2i}+2^i)}$	New	–	–	–	New I $(i=1, c=u^{31}, b=1)$ New II $(i=3, , c=u^{31}, b=1)$	–
4	$x(x^{2^i} + x^q + cx^{2^i q})$ $+x^{2^i}(c^q x^q + sx^{2^i q}) + x^{(2^i+1)q}$	$N^\circ 3$	–	New	–	$N^\circ 3$: Case I $(i=1, c=u^3, s=u)$ $N^\circ 3$: Case II $(i=3, c=u^3, s=w)$	–
5	$x^3 + a^{-1}\mathrm{tr}_n(a^3x^9)$	Gold $(a=1)$ $N^\circ 3$ $(a=u)$	New	New I $(a=1)$ New II $(a=u)$	New	New I $(a=1)$ New II $(a=u)$	New
6	$x^3 + a^{-1}\mathrm{tr}_n^3(a^3x^9 + a^6x^{18})$	Gold $(a=1)$ $N^\circ 3$ $(a=u)$	–	–	New	–	–
7	$x^3 + a^{-1}\mathrm{tr}_n^3(a^6x^{18} + a^{12}x^{36})$	Gold $(a=1)$ $N^\circ 3$ $(a=u)$	–	–	New	–	–
8	$ux^{2^s+1} + u^{2^k} x^{2^{-k}+2^{k+s}} +$ $vx^{2^{-k}+1} + wu^{2^k+1} x^{2^s+2^{k+s}}$, $v=0, w\neq 0$	New	–	–	–	–	–
9	$ux^{2^s+1} + u^{2^k} x^{2^{-k}+2^{k+s}} +$ $vx^{2^{-k}+1} + wu^{2^k+1} x^{2^s+2^{k+s}}$, $v\neq 0, w=0$	$N^\circ 8$	–	–	–	–	–
10	$ux^{2^s+1} + u^{2^k} x^{2^{-k}+2^{k+s}} +$ $vx^{2^{-k}+1} + wu^{2^k+1} x^{2^s+2^{k+s}}$, $v\neq 0, w\neq 0$	$N^\circ 8$	–	–	–	–	–
11	$(x + x^{2^m})^{2^k+1} +$ $u^{(2^n-1)/(2^m-1)}(ux + u^{2^m}x^{2^m})^{(2^k+1)2^i} +$ $u(x + x^{2^m})(ux + u^{2^m}x^{2^m})$	–	–	New $(i=2)$ $N^\circ 4$ $(i=0)$	–	–	–

* : For the conditions for each family, please reference table 2;
– : Doesn't exist;
New : CCZ-inequivalent with other infinite families.

Table 2: Known classes of quadratic APN polynomials CCZ-inequivalent to APN monomials over $\mathbb{F}_{2^n}$ (u is primitive in $\mathbb{F}_{2^n}^*$)

No.	Functions	Conditions
1-2 [3]	$x^{2^s+1} + u^{2^k-1} x^{2^{ik}+2^{mk+s}}$	$n = pk$, $\gcd(k,3) = \gcd(s,3k) = 1$, $p \in \{3,4\}$, $i = sk \bmod p$, $m = p - i$, $n \geq 12$
3 [2]	$x^{2^{2i}+2^i} + bx^{q+1} + cx^{q(2^{2i}+2^i)}$	$q = 2^m$, $n = 2m$, $\gcd(i,m) = 1$, $\gcd(2^i+1, q+1) \neq 1, cb^q + b \neq 0$, $c \notin \{\lambda^{(2^i+1)(q-1)}, \lambda \in \mathbb{F}_{2^n}\}, c^{q+1} = 1$
4 [2]	$x(x^{2^i} + x^q + cx^{2^i q})$ $+x^{2^i}(c^q x^q + sx^{2^i q}) + x^{(2^i+1)q}$	$q = 2^m, n = 2m, \gcd(i,m) = 1$, $c \in \mathbb{F}_{2^n}, s \in \mathbb{F}_{2^n} \setminus \mathbb{F}_q$, $X^{2^i+1} + cX^{2^i} + c^q X + 1$ is irreducible over $\mathbb{F}_{2^n}$
5 [4, 5]	$x^3 + a^{-1}\mathrm{tr}_n(a^3x^9)$	$a \neq 0$
6 [5]	$x^3 + a^{-1}\mathrm{tr}_n^3(a^3x^9 + a^6x^{18})$	$3\|n$, $a \neq 0$
7 [5]	$x^3 + a^{-1}\mathrm{tr}_n^3(a^6x^{18} + a^{12}x^{36})$	$3\|n$, $a \neq 0$
8-10 [1]	$ux^{2^s+1} + u^{2^k} x^{2^{-k}+2^{k+s}} +$ $vx^{2^{-k}+1} + wu^{2^k+1}x^{2^s+2^{k+s}}$	$n = 3k$, $\gcd(k,3) = \gcd(s,3k) = 1$, $v, w \in \mathbb{F}_{2^k}, vw \neq 1$, $3\|(k+s)$
11 [6]	$(x + x^{2^m})^{2^k+1} +$ $u^{(2^n-1)/(2^m-1)}(ux + u^{2^m}x^{2^m})^{(2^k+1)2^i} +$ $u(x + x^{2^m})(ux + u^{2^m}x^{2^m})$	$m \geq 2$, $2\|m$, $n = 2m$, $\gcd(k,m) = 1$, i is even

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