Table 1: Classification of Quadratic APN Trinomials (CCZ-inequivalent with infinite monomial families) in Small Dimensions with Coefficients as 1

| $n$ | $N^{\circ}$ | Functions | Families from Tables 5 | Relation to [6] |
| :---: | :---: | :---: | :---: | :---: |
| 6 | - | - | - | - |
| 7 | 7.1 | $x^{20}+x^{6}+x^{3}$ | - | Table $7: N^{\circ} 8.1$ |
|  | 7.2 | $x^{34}+x^{18}+x^{5}$ | - | $7: N^{\circ} 2.1$ |
| 8 | 8.1 | $x^{72}+x^{6}+x^{3}$ | $N^{\circ} 5$ | Table $9: N^{\circ} 1.3$ |
|  | 8.2 | $x^{72}+x^{36}+x^{3}$ | - | $9: N^{\circ} 1.4$ |
| 9 | - | - | - | - |
| 10 | - | - | - | - |
| 11 | - | - | - | - |

Table 2: Classification of Quadratic APN Quadrinomials (CCZ-inequivalent with infinite monomial families) in Small Dimensions with Coefficients as 1

| $n$ | $N^{\circ}$ | Functions | Families from Tables 5 | Relation to [6] |
| :---: | :---: | :---: | :---: | :---: |
| 6 | - | - | - | - |
| 7 | 7.1 | $x^{72}+x^{40}+x^{12}+x^{3}$ | - | Table $7: N^{\circ} 12.1$ |
|  | 7.2 | $x^{33}+x^{17}+x^{12}+x^{3}$ | - | $7: N^{\circ} 10.1$ |
|  | 7.3 | $x^{34}+x^{33}+x^{10}+x^{3}$ | - | $7: N^{\circ} 2.2$ |
|  | 7.4 | $x^{66}+x^{34}+x^{20}+x^{3}$ | - | $7: N^{\circ} 11.1$ |
|  | 7.5 | $x^{68}+x^{18}+x^{5}+x^{3}$ | - | $7: N^{\circ} 8.1$ |
|  | 7.6 | $x^{66}+x^{18}+x^{9}+x^{3}$ | - | $7: N^{\circ} 9.1$ |
| 8 | - | - | - | - |
| 9 | - | - | - | - |
| 10 | - | - | - | - |
| 11 | - | - | - | - |

Table 3: Classification of Quadratic APN Pentanomials (CCZ-inequivalent with infinite monomial families) in Small Dimensions with Coefficients as 1

| $n$ | $N^{\circ}$ | Functions | Families from Tables 5 | Relation to $[6]$ |
| :---: | :---: | :---: | :---: | :---: |
| 6 | - | - | - | - |
| 7 | 7.1 | $x^{68}+x^{40}+x^{24}+x^{6}+x^{3}$ | - | Table $7: N^{\circ} 13.1$ |
|  | 7.2 | $x^{65}+x^{20}+x^{18}+x^{6}+x^{3}$ | $N^{\circ} 5$ | $7: N^{\circ} 1.2$ |
|  | 7.3 | $x^{40}+x^{34}+x^{18}+x^{10}+x$ | - | $7: N^{\circ} 12.1$ |
|  | 7.4 | $x^{48}+x^{40}+x^{10}+x^{9}+x^{3}$ | - | $7: N^{\circ} 2.1$ |
|  | 7.5 | $x^{33}+x^{9}+x^{6}+x^{5}+x^{3}$ | - | $7: N^{\circ} 11.1$ |
|  | 7.6 | $x^{40}+x^{36}+x^{34}+x^{24}+x^{3}$ | - | $7: N^{\circ} 10.1$ |
|  | 7.7 | $x^{24}+x^{10}+x^{9}+x^{6}+x^{3}$ | - | $7: N^{\circ} 2.2$ |
|  | 7.8 | $x^{65}+x^{36}+x^{20}+x^{17}+x^{3}$ | - | $7: N^{\circ} 14.1$ |
|  | 7.9 | $x^{40}+x^{33}+x^{17}+x^{5}+x^{3}$ | $-N^{\circ} 8.1$ |  |
|  | 7.10 | $x^{36}+x^{33}+x^{18}+x^{9}+x^{5}$ | - | $7: N^{\circ} 10.2$ |
| 8 | 8.1 | $x^{36}+x^{33}+x^{9}+x^{6}+x^{3}$ | - | Table $9: N^{\circ} 1.4$ |
|  | 8.2 | $x^{72}+x^{66}+x^{12}+x^{6}+x^{3}$ | $N^{\circ} 5$ | $9: N^{\circ} 1.3$ |
|  | 8.3 | $x^{130}+x^{66}+x^{40}+x^{12}+x^{3}$ | - | $9: N^{\circ} 6.1$ |
|  | 8.4 | $x^{66}+x^{40}+x^{18}+x^{5}+x^{3}$ | - | $9: N^{\circ} 5.1$ |
| 9 | - |  | - | - |
| 10 | - |  | - | - |
|  | 11.1 | $x^{12}+x^{10}+x^{9}+x^{5}+x^{3}$ | - | - |
|  | 11.2 | $x^{258}+x^{257}+x^{18}+x^{17}+x^{3}$ | - | - |
| 11 | 11.3 | $x^{96}+x^{66}+x^{34}+x^{33}+x^{3}$ | - | - |
|  | 11.4 | $x^{80}+x^{68}+x^{65}+x^{17}+x^{5}$ | - | - |
|  | 11.5 | $x^{260}+x^{257}+x^{36}+x^{33}+x^{5}$ | - | - |

Table 4: Classification of Quadratic APN Hexanomial (CCZ-inequivalent with infinite monomial families) in Small Dimensions with Coefficients as 1

| $n$ | $N^{\circ}$ | Functions | Families from Tables 5 | Relation to [6] |
| :---: | :---: | :---: | :---: | :---: |
| 6 | - | - | - | - |
| 7 | 7.1 | $x^{34}+x^{33}+x^{12}+x^{6}+x^{5}+x^{3}$ | - | Table $7: N^{\circ} 14.2$ |
|  | 7.2 | $x^{40}+x^{24}+x^{20}+x^{9}+x^{5}+x^{3}$ | - | $7: N^{\circ} 14.1$ |
|  | 7.3 | $x^{33}+x^{24}+x^{20}+x^{18}+x^{12}+x^{3}$ | - | $7: N^{\circ} 12.1$ |
|  | 7.4 | $x^{24}+x^{17}+x^{12}+x^{10}+x^{6}+x^{3}$ | - | $7: N^{\circ} 2.1$ |
|  | 7.5 | $x^{40}+x^{34}+x^{18}+x^{17}+x^{5}+x^{3}$ | $N^{\circ} 5$ | $7: N^{\circ} 1.2$ |
|  | 7.6 | $x^{48}+x^{40}+x^{18}+x^{10}+x^{5}+x^{3}$ | - | $7: N^{\circ} 11.1$ |
|  | 7.7 | $x^{40}+x^{12}+x^{10}+x^{9}+x^{5}+x^{3}$ | - | $7: N^{\circ} 2.2$ |
|  | 7.8 | $x^{34}+x^{24}+x^{10}+x^{9}+x^{6}+x^{3}$ | - | $7: N^{\circ} 9.1$ |
|  | 7.9 | $x^{34}+x^{33}+x^{20}+x^{17}+x^{10}+x^{3}$ | - | $7: N^{\circ} 13.1$ |
|  | 7.10 | $x^{36}+x^{33}+x^{24}+x^{9}+x^{6}+x^{3}$ | - | $7: N^{\circ} 10.1$ |
|  | 7.11 | $x^{40}+x^{36}+x^{20}+x^{10}+x^{5}+x^{3}$ | - | $7: N^{\circ} 10.2$ |
|  | 7.12 | $x^{36}+x^{34}+x^{20}+x^{10}+x^{9}+x^{3}$ | - | $7: N^{\circ} 8.1$ |
| 8 | 8.1 | $x^{68}+x^{34}+x^{17}+x^{12}+x^{9}+x^{3}$ | - | Table $9: N^{\circ} 5.1$ |
|  | 8.2 | $x^{72}+x^{40}+x^{34}+x^{20}+x^{12}+x^{3}$ | - | $9: N^{\circ} 6.1$ |
|  | 8.3 | $x^{72}+x^{66}+x^{34}+x^{18}+x^{10}+x^{5}$ | $N^{\circ} 5$ | $9: N^{\circ} 4.1$ |
| 9 | - | - | - | - |
| 10 | - | - | - | - |
| 11 | - | - | - | - |

Table 5: Known classes of quadratic APN polynomials CCZ-inequivalent to APN monomials on $\mathbb{F}_{2^{n}}\left(u\right.$ is primitive in $\left.\mathbb{F}_{2^{n}}^{*}\right)$

| No. | Functions | Conditions |
| :---: | :---: | :---: |
| $\begin{gathered} 1-2 \\ {[3]} \end{gathered}$ | $x^{2^{s}+1}+u^{2^{k}-1} x^{2^{i k}+2^{m k+s}}$ | $\begin{gathered} n=p k, \operatorname{gcd}(k, 3)=\operatorname{gcd}(s, 3 k)=1, \\ p \in\{3,4\}, i=s k \bmod p, m=p-i \\ n \geq 12 \end{gathered}$ |
| $\begin{gathered} 3 \\ {[2]} \end{gathered}$ | $x^{2^{2 i}+2^{i}}+b x^{q+1}+c x^{q\left(2^{2 i}+2^{i}\right)}$ | $\begin{gathered} q=2^{m}, n=2 m, \operatorname{gcd}(i, m)=1, \\ \operatorname{gcd}\left(2^{i}+1, q+1\right) \neq 1, c b^{q}+b \neq 0, \\ c \notin\left\{\lambda^{\left(2^{i}+1\right)(q-1)}, \lambda \in \mathbb{F}_{2^{n}}\right\}, c^{q+1}=1 \end{gathered}$ |
| $\begin{gathered} 4 \\ {[2]} \end{gathered}$ | $\begin{gathered} x\left(x^{2^{i}}+x^{q}+c x^{2^{i} q}\right) \\ +x^{2^{i}}\left(c^{q} x^{q}+s x^{2^{i} q}\right)+x^{\left(2^{i}+1\right) q} \end{gathered}$ | $\begin{gathered} q=2^{m}, n=2 m, \operatorname{gcd}(i, m)=1, \\ c \in \mathbb{F}_{2^{n}}, s \in \mathbb{F}_{2^{n}} \backslash \mathbb{F}_{q}, \\ X^{2^{i}+1}+c X^{2^{i}}+c^{q} X+1 \end{gathered}$ <br> is irreducible over $\mathbb{F}_{2} n$ |
| $\begin{gathered} 5 \\ {[4,5]} \end{gathered}$ | $x^{3}+a^{-1} \operatorname{tr}_{n}\left(a^{3} x^{9}\right)$ | $a \neq 0$ |
| $\begin{gathered} 6 \\ {[5]} \end{gathered}$ | $x^{3}+a^{-1} \operatorname{tr}_{n}^{3}\left(a^{3} x^{9}+a^{6} x^{18}\right)$ | $3 \mid n, a \neq 0$ |
| $\begin{gathered} \hline 7 \\ {[5]} \end{gathered}$ | $x^{3}+a^{-1} \operatorname{tr}_{n}^{3}\left(a^{6} x^{18}+a^{12} x^{36}\right)$ | $3 \mid n, a \neq 0$ |
| $\begin{gathered} 8-10 \\ {[1]} \end{gathered}$ | $\begin{gathered} u x^{2^{s}+1}+u^{2^{k}} x^{2^{-k}+2^{k+s}}+ \\ v x^{2^{-k}+1}+w u^{2^{k}+1} x^{2^{s}+2^{k+s}} \end{gathered}$ | $\begin{gathered} n=3 k, \operatorname{gcd}(k, 3)=\operatorname{gcd}(s, 3 k)=1, \\ v, w \in \mathbb{F}_{2^{k}}, v w \neq 1, \\ 3 \mid(k+s) \end{gathered}$ |
| $\begin{aligned} & 11 \\ & {[7]} \end{aligned}$ | $\begin{gathered} \left(x+x^{2^{m}}\right)^{2^{k}+1}+ \\ u^{\left(2^{n}-1\right) /\left(2^{m}-1\right)}\left(u x+u^{2^{m}} x^{2^{m}}\right)^{\left(2^{k}+1\right) 2^{i}}+ \\ u\left(x+x^{2^{m}}\right)\left(u x+u^{2^{m}} x^{2^{m}}\right) \\ \hline \end{gathered}$ | $\begin{gathered} m \geq 2,2 \mid m, n=2 m \\ \operatorname{gcd}(k, m)=1, i \text { is even } \end{gathered}$ |

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